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# MATHEMATICAL GEOMETRY BASED FILTERS

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**Abstract: -**

Nonlinear two-dimensional image restoration filter structure is introduced in this work. Nonlinear prediction structure is proposed using nonlinear element depending on eyes visually phenomena of noise detection. Noise detection based mathematical geometry is introduced. The mathematical geometry procedure used to calculate the triangular distribution of image data. Nonlinear filter is applied to the noisy detected pixels. Filter stability is demanded in this structure. The stability is insured in case of recursive or non-recursive connection. Impulse noise recovery is guaranteed in this filter. The advantage of this filter is reservation of textures and keeping unchanged of fine details. Median based filters are proposed for noise recovery.

**Keywords**: Digital Filters, Image Processing, Gamma Compensation

## 1. Introduction

Nonlinear filters show a preferred solution of nonlinear data processing problem solving [l]. Most proposed nonlinear filters were non-recursive. Recursive filter has more efficient result concerning its arithmetic operations needed during its implementations [2, 3]. Stability was the important risk in designing such filter structure. The powerful concept to solve the stability problems is passive digital systems [4, 5]. Very useful concept is shown during Image Compression systems and Signal Coding Prediction [6- 8]. This work applies a design of nonlinear digital filters to process colour images. Nonlinear spatial detection of noise and prediction of lost data is suggested. Noise detection and prediction structure depends on geometrical test of local samples based on mathematical image geometry structure is explained. The advantage of this work is nonlinear filters performance improvement by additional prediction path.

New original structure that precedes the predicted signal including nonlinear filter is presented. A new structure based on mathematical geometry for noise detection is presented. The new structure depends on examining the triangular distribution around the processed pixel. Reducing filter degradation of fine details and thin lines caused by classic nonlinear filters is the main aim in this new structure.

## 2. Mathematical Geometry

Introducing mathematics as application topics is a major matter of engineering topics when it depends on periodical signals. Choosing a suitable mathematical part in geometry is the difficult matter.

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- Geometry origins in the pre-Greek antiquity where most mathematics measurement was considered as geometry.
- Felix Klein's Erlanger program proposed to classify mathematics and especially geometry algebraically by group of isomorphism.
- Today, geometry is mainly used in the sense of various topology and differentiable manifolds.

Differentiable manifolds are discrete that does not exclude finite geometries. Geometry and geometrical generations of mathematics are useless to put boundaries in a specific mathematical field. Any essential field of mathematics influences the other essential fields. This fact is indicate that any ordered set of topics {"algebra", "analysis", "geometry", "probability theory" or "number theory"} and form pairs such as {geometric, analysis} defines itself as a section of mathematics. Riemannian manifolds, Algebraic geometry, Simplistic geometry, the study of varieties as algebraic manifolds, the study of simplistic manifolds, differential geometry on fiber bundles, Geometry of Gauge fields, geometry on spaces with fractal dimensions, Spectral Geometry, the spectral theory of differential operators on a manifold. Non-commutative Geometry, foliations and discrete manifolds is the other sections of geometry. The importance of geometry is that many classical physical theories can be described in a purely geometrical way [9]. Examples are classical mechanics, electromagnetism and other gauge field theories, general relativity or the standard model in particle physics. In mathematical geometry, the problem is considered in figure 1.

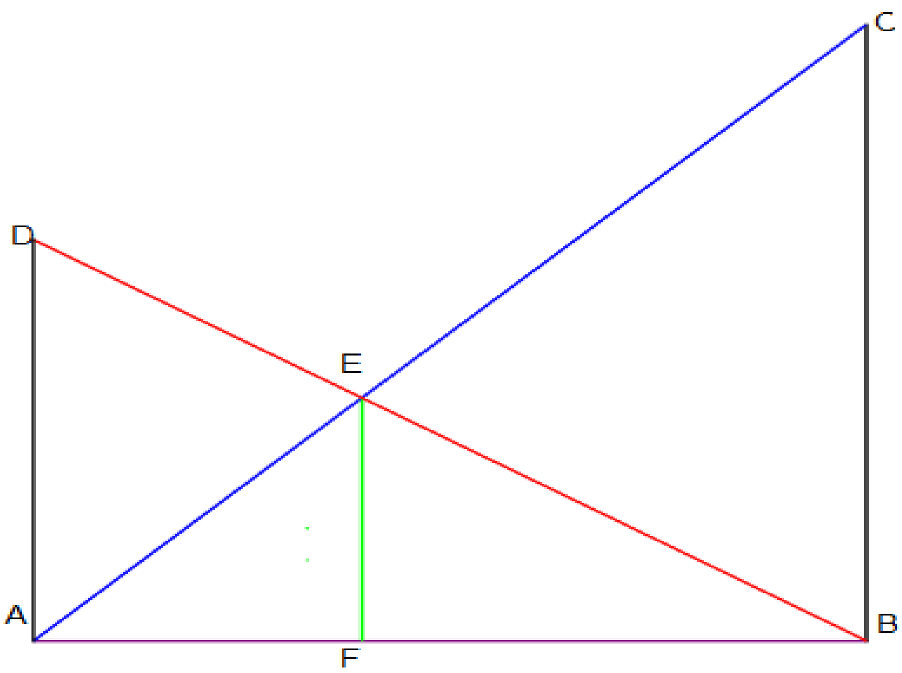


Fig. 1: Mathematical Geometry

Now we should calculate the length of EF, we denote the following

The point of E has the coordinate $(E_x, E_y)$

$E_y$ Is the length of EF.

The point of A has the coordinate $(A_x, A_y)$

The point of B has the coordinate $(B_x, B_y)$

The point of C has the coordinate $(C_x, C_y)$

The point of D has the coordinate $(D_x, D_y)$

Note that $C_x = B_x$ and $A_x = D_x$
After some derivations we have:

$$E_x = \frac{(B_x)^2 + (A_x)^2 - (A_x - B_x)^2}{B_x + A_x}$$

$$E_y = \frac{B_x}{B_x - A_x}\left\{\frac{(B_x)^2 + (A_x)^2 - (A_x - B_x)^2}{B_x + A_x} - B_x\right\} + B_x$$

## 3. FILTER PROCESSING

Referring figure 1, the interval within point A and B is considered as the 9 samples from 3x3 window centred at each pixel. Using Monte Carlo simulation to set the minimum value of the samples at point A. point B is set as the maximum value of the samples. Now, exact expected samples value can be calculated. The key is in understanding how the probability density function associated with triangular distributions work. The probability density function is shown in figure 2 and defined as follows:

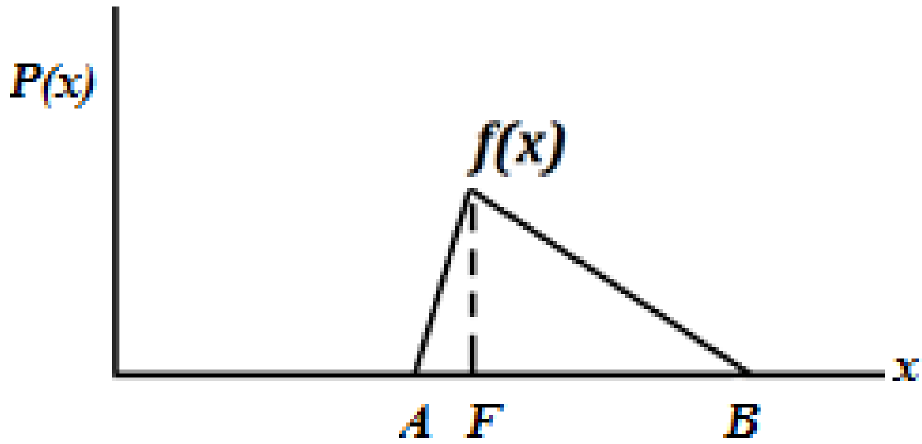


Fig 2: Triangular Distribution Concept.

$$f(x) = \begin{cases} \dfrac{2(x-A)}{(B-A)(F-A)} & \text{for } A \le x \le F \\ \dfrac{2(B-x)}{(B-A)(B-F)} & \text{for } F \le x \le B \\ 0 & \text{otherwise} \end{cases}$$

For this simulation, (A) is the minimum value of samples (x) in a given image, (B) is the maximum value, and (F) is the mode value. The peak, or maximum value of the function, is 2/(A-B) at sample F. The expected value of image sample is the sum of two integrals:

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The definite integral of from A to F is $\int_A^F x.\,f(x)\,dx$

The definite integral from F to B is $\int_F^B x.\,f(x)\,dx$

Processed pixel lay in the simulated triangle is considered as uncorrupted pixels. Corrupted parts of pixels are lying outside the triangle. Corrupted parts of pixels are considered to have a probability function of zero value. Median filter is applied to the corrupted pixels only. The median filter is processed in RGB color space due to its linear transformation [10]. In some modifications of this simple filter it is easy to expand this application to a vector filter. The described filters process textures and small details much better than median filters of different kind.

## 4. Results and Conclusion

Baboon, Lena, Boats, and Pepper are selected as a test images. Impulsive noise is applied uniformly with amplitude (0-255) to the RGB components of test images randomly with probability of p. Test images are filtered using median filter. A comparison is done by applying median filter to the detected parts of the image that corrupted by noise with prediction structure only.

Filters efficiency is proved by calculation of peak to noise ratio (PSNR) for objective quality. Filter performance with prediction (PF) is superior to that of the median filter (MF) in subjective quality also. Fine details and texture are preserved properly in filter output by use of detection procedure. Table (1) shows noticeable increasing in [dB] when the prediction procedure is used. Better improvement is obtained when recursive filter (RPF) is applied with comparison against recursive median filter (RMF) as observed in Table 2. Figure 3 is focusing a better preservation of fine textures with the proposed structure application. This illustrates the advantages by increasing the efficiency of impulsive noise rejection (Table 1 and 2).

Table 1: Impulse noise rejection output for non-recursive filters.

| Input Image | PSNR[dB] for impulse noise *p*=5% | | | PSNR [dB] for impulse noise *p*=10% | | |
|---|---|---|---|---|---|---|
| | With noise | MF | PF | With noise | MF | PF |
| Baboon | 13.65 | 15.66 | 21.99 | 10.66 | 15.48 | 18.94 |
| Boats | 13.51 | 21.65 | 26.69 | 10.50 | 21.01 | 23.61 |
| Lena | 13.49 | 23.53 | 28.62 | 10.55 | 22.88 | 25.41 |
| Peppers | 13.31 | 25.25 | 29.38 | 10.43 | 24.40 | 25.92 |

Table 2: Impulse noise rejection output for recursive filter.

| Input Image | PSNR[dB] for impulse noise *p*=5% | | | PSNR [dB] for impulse noise *p*=10% | | |
|---|---|---|---|---|---|---|
| | Noisy | RMF | RPF | Noisy | RMF | RPF |
| Baboon | 13.72 | 15.82 | 22.02 | 10.71 | 15.62 | 29.211 |
| Boats | 13.54 | 21.80 | 27.04 | 10.53 | 21.31 | 23874 |
| Lena | 13.54 | 23.72 | 28.93 | 10.56 | 23.210 | 25.51 |
| Peppers | 13.35 | 25.57 | 29.32 | 10.39 | 24.95 | 26.79 |

Original Image

Corrupted Image by impulse noise

Median filter output

Proposed filter output

Fig. 3: Impulse noise rejection using proposed recursive filter (RPF) compared with recursive median filter (RMF) with probability of $p$=10%.

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