INTERNATIONAL JOURNAL OF RESEARCH IN COMPUTER APPLICATIONS AND ROBOTICS
ISSN 2320-7345

USING MAPLE TO STUDY TWO TYPES OF INTEGRALS

Chii-Huei Yu

Department of Management and Information, Nan Jeon Institute of Technology, Tainan City, Taiwan
chiihuei@mail.njtc.edu.tw

Abstract

This study uses the mathematical software Maple for the auxiliary tool to evaluate two types of integrals. We can obtain the infinite series forms of these two types of integrals mainly using geometric series and integration term by term. At the same time, we provide some integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

Keywords: integrals; infinite series forms; geometric series; integration term by term; Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, [1]-[7] can be adopted as references.

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems, including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we study the evaluation of the following two types of indefinite integrals which are not easy to obtain their answers by using the methods mentioned above.

$$\int \frac{e^{cx}\sin(\lambda x+\beta)}{a+b\cos(\lambda x+\beta)}dx \tag{1}$$

$$\int \frac{e^{cx}[a+\sqrt{a^2-b^2}+b\cos(\lambda x+\beta)]}{a+b\cos(\lambda x+\beta)}dx \tag{2}$$

, where λ,β,a,b,c are real numbers, $b,c\neq 0$, and $a>|b|$. We can obtain the infinite series forms of these two types of integrals mainly using geometric series and integration term by term; these are the major results in this study (i.e., Theorems 1, 2). In addition, we obtain two corollaries from these two theorems. As for the study of related integral problems can refer to [8]-[16]. On the other hand, we provide some integrals to determine their infinite series forms practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce one notation and two formulas used in this study.

Notation.

Let $z=a+ib$ be a complex number, where $i=\sqrt{-1}$, a,b are real numbers. We denote a the real part of z by $\mathrm{Re}(z)$, and b the imaginary part of z by $\mathrm{Im}(z)$.

Euler's formula.

$e^{iy}=\cos y+i\sin y$, where y is any real number.

DeMoivre's formula.

$(\cos y+i\sin y)^n=\cos ny+i\sin ny$, where n is any integer, y is any real number.

Next, we introduce two important theorems used in this paper.

Geometric series.

$\sum_{k=0}^{\infty} z^k=\frac{1}{1-z}$, where z is a complex number, and $|z|<1$.

Integration term by term. ([17])

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an inteval I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n=\sum_{n=0}^{\infty}\int_I g_n$.

The following is the first major result of this study, we determine the infinite series form of indefinite integral (1).

Theorem 1. Suppose λ,β,a,b,c are real numbers, $b,c\neq 0$, $a>|b|$, and C is a constant. Then the indefinite integral

$$\int \frac{e^{cx}\sin(\lambda x+\beta)}{a+b\cos(\lambda x+\beta)}dx$$

$$= \frac{-2}{b} \cdot e^{cx} \sum_{k=1}^{\infty} \left(-\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} \right)^{k} \frac{1}{c^{2}+\lambda^{2}k^{2}} [c \sin(\lambda kx + \beta k) - \lambda k \cos(\lambda kx + \beta k)] + C \tag{3}$$

Proof. Let $r = \frac{1}{2}(\sqrt{a+b}+\sqrt{a-b})$, $s = \frac{1}{2}(\sqrt{a+b}-\sqrt{a-b})$, then

$$\frac{e^{cx}\sin(\lambda x+\beta)}{a+b\cos(\lambda x+\beta)}$$

$$= e^{cx} \cdot \frac{\sin(\lambda x+\beta)}{r^{2}+2rs\cos(\lambda x+\beta)+s^{2}}$$

$$= e^{cx} \cdot \frac{\frac{1}{s^{2}} \cdot \sin(\lambda x+\beta)}{\left(\frac{r}{s}+\cos(\lambda x+\beta)\right)^{2}+\sin^{2}(\lambda x+\beta)}$$

$$= e^{cx} \cdot \mathrm{Im}\left[\frac{\frac{1}{s^{2}}\left(\frac{r}{s}+\cos(\lambda x+\beta)+i\sin(\lambda x+\beta)\right)}{\left(\frac{r}{s}+\cos(\lambda x+\beta)+i\sin(\lambda x+\beta)\right)\left(\frac{r}{s}+\cos(\lambda x+\beta)-i\sin(\lambda x+\beta)\right)}\right]$$

$$= \frac{1}{s^{2}} e^{cx} \cdot \mathrm{Im}\left(\frac{1}{\frac{r}{s}+z}\right) \quad (\text{where } z = e^{-i(\lambda x+\beta)})$$

$$= \frac{1}{rs} e^{cx} \cdot \mathrm{Im}\left(\frac{1}{1+\frac{s}{r}z}\right)$$

$$= \frac{1}{rs} e^{cx} \cdot \mathrm{Im}\left[\sum_{k=0}^{\infty}\left(-\frac{sz}{r}\right)^{k}\right] \quad (\text{because } \left|-\frac{sz}{r}\right| = \left|\frac{s}{r}\right| < 1 \text{, we can use geometric series})$$

$$= \frac{1}{rs} e^{cx} \cdot \mathrm{Im}\left[\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^{k} e^{-i(\lambda kx+\beta k)}\right] \quad (\text{by DeMoivre's formula})$$

$$= \frac{-2}{b} e^{cx} \cdot \sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^{k} \sin(\lambda kx + \beta k) \quad (\text{by Euler's formula}) \tag{4}$$

Therefore, the indefinite integral

$$\int \frac{e^{cx}\sin(\lambda x+\beta)}{a+b\cos(\lambda x+\beta)}dx$$

$$=\frac{-2}{b}\int e^{cx}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k\sin(\lambda kx+\beta k)dx$$

$$=\frac{-2}{b}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k\int e^{cx}\sin(\lambda kx+\beta k)dx \quad \text{(using integration term by term)}$$

$$=\frac{-2}{b}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Im}\left[\int e^{cx+i(\lambda kx+\beta k)}dx\right]$$

$$=\frac{-2}{b}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Im}\left[\int e^{(c+i\lambda k)x+i\beta k}dx\right]$$

$$=\frac{-2}{b}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Im}\left[\frac{1}{c+i\lambda k}\cdot e^{(c+i\lambda k)x+i\beta k}\right]$$

$$=\frac{-2}{b}\cdot\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Im}\left[\frac{(c-i\lambda k)}{c^2+\lambda^2k^2}\cdot e^{cx+i(\lambda kx+\beta k)}\right]$$

$$=\frac{-2}{b}\cdot e^{cx}\sum_{k=1}^{\infty}\left(-\frac{s}{r}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\sin(\lambda kx+\beta k)-\lambda k\cos(\lambda kx+\beta k)]+C$$

$$=\frac{-2}{b}\cdot e^{cx}\sum_{k=1}^{\infty}\left(-\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\sin(\lambda kx+\beta k)-\lambda k\cos(\lambda kx+\beta k)]+C$$

■

Using Theorem 1, we immediately obtain the following result.

Corollary 1. In Theorem 1, if $c<0$, and let t be any real number, then the improper integral

$$\int_t^{\infty} \frac{e^{cx}\sin(\lambda x+\beta)}{a+b\cos(\lambda x+\beta)}dx$$

$$=\frac{2}{b}\cdot e^{ct}\sum_{k=1}^{\infty}\left(-\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\sin(\lambda kt+\beta k)-\lambda k\cos(\lambda kt+\beta k)] \qquad (5)$$

Next, we find the infinite series form of indefinite integral (2).

Theorem 2. If the assumptions are the same as Theorem 1, then the indefinite integral

$$\int \frac{e^{cx}[a+\sqrt{a^2-b^2}+b\cos(\lambda x+\beta)]}{a+b\cos(\lambda x+\beta)}dx$$

$$= 2 \cdot e^{cx} \sum_{k=0}^{\infty} \left(- \frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}} \right)^k \frac{1}{c^2 + \lambda^2 k^2} [c \cos(\lambda kx + \beta k) + \lambda k \sin(\lambda kx + \beta k)] + C \quad (6)$$

Proof. Also let $r = \frac{1}{2}(\sqrt{a+b} + \sqrt{a-b})$, $s = \frac{1}{2}(\sqrt{a+b} - \sqrt{a-b})$, then

$$\frac{e^{cx}[a + \sqrt{a^2 - b^2} + b\cos(\lambda x + \beta)]}{a + b\cos(\lambda x + \beta)}$$

$$= be^{cx} \cdot \frac{\frac{r}{s} + \cos(\lambda x + \beta)}{r^2 + 2rs\cos(\lambda x + \beta) + s^2}$$

$$= \frac{b}{s^2} e^{cx} \cdot \frac{\frac{r}{s} + \cos(\lambda x + \beta)}{\left(\frac{r}{s} + \cos(\lambda x + \beta) \right)^2 + \sin^2(\lambda x + \beta)}$$

$$= \frac{b}{s^2} e^{cx} \cdot \mathrm{Re} \left[\frac{\frac{r}{s} + \cos(\lambda x + \beta) + i\sin(\lambda x + \beta)}{\left(\frac{r}{s} + \cos(\lambda x + \beta) + i\sin(\lambda x + \beta) \right)\left(\frac{r}{s} + \cos(\lambda x + \beta) - i\sin(\lambda x + \beta) \right)} \right]$$

$$= \frac{b}{s^2} e^{cx} \cdot \mathrm{Re} \left(\frac{1}{\frac{r}{s} + z} \right) \quad (\text{where } z = e^{-i(\lambda x + \beta)})$$

$$= \frac{b}{rs} e^{cx} \cdot \mathrm{Re} \left(\frac{1}{1 + \frac{s}{r} z} \right)$$

$$= 2e^{cx} \cdot \mathrm{Re} \left[\sum_{k=0}^{\infty} \left(- \frac{sz}{r} \right)^k \right] \quad \text{(by geometric series)}$$

$$= 2e^{cx} \cdot \mathrm{Re} \left[\sum_{k=0}^{\infty} \left(- \frac{s}{r} \right)^k e^{-i(\lambda kx + \beta k)} \right] \quad \text{(by DeMoivre's formula)}$$

$$= 2e^{cx} \cdot \sum_{k=0}^{\infty} \left(- \frac{s}{r} \right)^k \cos(\lambda kx + \beta k) \quad \text{(by Euler's formula)} \quad (7)$$

Thus, the indefinite integral

$$\int \frac{e^{cx}[a+\sqrt{a^2-b^2}+b\cos(\lambda x+\beta)]}{a+b\cos(\lambda x+\beta)}dx$$

$$=2\cdot\int e^{cx}\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k\cos(\lambda kx+\beta k)dx$$

$$=2\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k\int e^{cx}\cos(\lambda kx+\beta k)dx \qquad \text{(by integration term by term)}$$

$$=2\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Re}\left[\int e^{cx+i(\lambda kx+\beta k)}dx\right]$$

$$=2\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Re}\left[\int e^{(c+i\lambda k)x+i\beta k}dx\right]$$

$$=2\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Re}\left[\frac{1}{c+i\lambda k}\cdot e^{(c+i\lambda k)x+i\beta k}\right]$$

$$=2\cdot\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k \operatorname{Re}\left[\frac{(c-i\lambda k)}{c^2+\lambda^2k^2}\cdot e^{cx+i(\lambda kx+\beta k)}\right]$$

$$=2\cdot e^{cx}\sum_{k=0}^{\infty}\left(-\frac{s}{r}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\cos(\lambda kx+\beta k)+\lambda k\sin(\lambda kx+\beta k)]+C$$

$$=2\cdot e^{cx}\sum_{k=0}^{\infty}\left(-\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\cos(\lambda kx+\beta k)+\lambda k\sin(\lambda kx+\beta k)]+C \qquad \blacksquare$$

By Theorem 2, we have the following result.

Corollary 2. In Theorem 2, if $c<0$, and let t be any real number, then the improper integral

$$\int_t^{\infty} \frac{e^{cx}[a+\sqrt{a^2-b^2}+b\cos(\lambda x+\beta)]}{a+b\cos(\lambda x+\beta)}dx$$

$$=-2\cdot e^{ct}\sum_{k=0}^{\infty}\left(-\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\right)^k \frac{1}{c^2+\lambda^2k^2}[c\cos(\lambda kt+\beta k)+\lambda k\sin(\lambda kt+\beta k)] \qquad (8)$$

3. Examples

In the following, aimed at the two types of integrals in this study, we propose four integrals and use Theorems 1, 2 and Corollaries 1, 2 to determine their infinite series forms. On the other hand, we use Maple to calculate the approximations of related definite integrals and their infinite series forms for verifying our answers.

Example 1. In Theorem 1, if we take $\lambda = 2, \beta = \pi/6, a = 5, b = 4, c = 3$, then the indefinite integral

$$\int \frac{e^{3x}\sin(2x+\pi/6)}{5+4\cos(2x+\pi/6)}dx = \frac{-1}{2}\cdot e^{3x}\sum_{k=1}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{9+4k^2}\left[3\sin\left(2kx+\frac{k\pi}{6}\right)-2k\cos\left(2kx+\frac{k\pi}{6}\right)\right]+C \tag{9}$$

Therefore, we can determine its definite integral from $x = \pi/6$ to $x = \pi/4$,

$$\int_{\pi/6}^{\pi/4} \frac{e^{3x}\sin(2x+\pi/6)}{5+4\cos(2x+\pi/6)}dx$$

$$= \frac{-1}{2}\cdot e^{3\pi/4}\sum_{k=1}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{9+4k^2}\left[3\sin\left(\frac{2k\pi}{3}\right)-2k\cos\left(\frac{2k\pi}{3}\right)\right]$$

$$+\frac{1}{2}\cdot e^{\pi/2}\sum_{k=1}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{9+4k^2}\left[3\sin\left(\frac{k\pi}{2}\right)-2k\cos\left(\frac{k\pi}{2}\right)\right] \tag{10}$$

Next, we employ Maple to verify the correctness of (10).

```
>evalf(int(exp(3*x)*sin(2*x+Pi/6)/(5+4*cos(2*x+Pi/6)),x=Pi/6..Pi/4),18);
```

0.477761126377698869

```
>evalf(-1/2*exp(3*Pi/4)*sum((-1/2)^k/(9+4*k^2)*(3*sin(2*k*Pi/3)-2*k*cos(2*k*Pi/3)),k=1..infinity)+1/2*
exp(Pi/2)*sum((-1/2)^k/(9+4*k^2)*(3*sin(k*Pi/2)-2*k*cos(k*Pi/2)),k=1..infinity),18);
```

0.477761126377698869 + 0. I

The above answer obtained by Maple appears I ($=\sqrt{-1}$), it is because Maple calculates by using special functions built in. The imaginary part is zero, so can be ignored.

Example 2. In Corollary 1, taking $\lambda = 3, \beta = \pi/4, a = 6, b = -4, c = -2, t = \pi/2$, we obtain the following improper integral from $x = \pi/2$ to $x = \infty$

$$\int_{\pi/2}^{\infty} \frac{e^{-2x}\sin(3x+\pi/4)}{6-4\cos(3x+\pi/4)}dx = -\frac{1}{2}\cdot e^{-\pi}\sum_{k=1}^{\infty}\left(\frac{\sqrt{10}-\sqrt{2}}{\sqrt{10}+\sqrt{2}}\right)^k \frac{1}{4+9k^2}\left[-2\sin\left(\frac{7k\pi}{4}\right)-3k\cos\left(\frac{7k\pi}{4}\right)\right] \tag{11}$$

We also use Maple to verify the correctness of (11).

```
>evalf(int(exp(-2*x)*sin(3*x+Pi/4)/(6-4*cos(3*x+Pi/4)),x=Pi/2..infinity),18);
```

0.000138579315603263011

```
>evalf(-1/2*exp(-Pi)*sum(((sqrt(10)-sqrt(2))/(sqrt(10)+sqrt(2)))^k/(4+9*k^2)*(-2*sin(7*k*Pi/4)-3*k*cos(7*
k*Pi/4)),k=1..infinity),18);
```

$0.000138579315603263014 - 2.91791997136612626\cdot 10^{-21}$ I

The above answer obtained by Maple also appears I, the imaginary part is very small, so can be ignored.

Example 3. In Theorem 2, if we take $\lambda = 4, \beta = 5\pi/4, a = 10, b = 8, c = 6$, then the indefinite integral

$$\int \frac{e^{6x}[16+8\cos(4x+5\pi/4)]}{10+8\cos(4x+5\pi/4)}dx$$

$$= 2\cdot e^{6x}\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{36+16k^2}\left[6\cos\left(4kx+\frac{5k\pi}{4}\right)+4k\sin\left(4kx+\frac{5k\pi}{4}\right)\right]+C \tag{12}$$

Thus, we obtain its definite integral from $x=\pi/8$ to $x=\pi/2$,

$$\int_{\pi/8}^{\pi/2} \frac{e^{6x}[16+8\cos(4x+5\pi/4)]}{10+8\cos(4x+5\pi/4)}dx$$

$$= 2\cdot e^{3\pi}\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{36+16k^2}\left[6\cos\left(\frac{5k\pi}{4}\right)+4k\sin\left(\frac{5k\pi}{4}\right)\right]$$

$$-2\cdot e^{3\pi/4}\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k \frac{1}{36+16k^2}\left[6\cos\left(\frac{7k\pi}{4}\right)+4k\sin\left(\frac{7k\pi}{4}\right)\right] \tag{13}$$

We employ Maple to verify the correctness of (13) as follows:

>evalf(int(exp(6*x)*(16+8*cos(4*x+5*Pi/4))/(10+8*cos(4*x+5*Pi/4)),x=Pi/8..Pi/2),18);

6300.26272255804089

>evalf(2*exp(3*Pi)*sum((-1/2)^k/(36+16*k^2)*(6*cos(5*k*Pi/4)+4*k*sin(5*k*Pi/4)),k=0..infinity)-2*exp(3*Pi/4)*sum((-1/2)^k/(36+16*k^2)*(6*cos(7*k*Pi/4)+4*k*sin(7*k*Pi/4)),k=0..infinity),18);

6300.26272255804091 − 0. I

The above answer obtained by Maple appears I, the imaginary part is zero, so can be ignored.

Example 4. In Corollary 2, if taking $\lambda=7, \beta=-\pi/6, a=13, b=5, c=-4, t=3\pi/4$, then we obtain the infinite series form of the improper integral from $x=3\pi/4$ to $x=\infty$,

$$\int_{3\pi/4}^{\infty} \frac{e^{-4x}[25+5\cos(7x-\pi/6)]}{13+5\cos(7x-\pi/6)}dx = -2\cdot e^{-3\pi}\sum_{k=0}^{\infty}\left(-\frac{1}{5}\right)^k \frac{1}{16+49k^2}\left[-4\cos\left(\frac{61k\pi}{12}\right)+7k\sin\left(\frac{61k\pi}{12}\right)\right] \tag{14}$$

In the following, we employ Maple to verify the correctness of (14).

>evalf(int(exp(-4*x)*(25+5*cos(7*x-Pi/6))/(13+5*cos(7*x-Pi/6)),x=3*Pi/4..infinity),18);

0.0000412183133260668986

>evalf(-2*exp(-3*Pi)*sum((-1/5)^k/(16+49*k^2)*(-4*cos(61*k*Pi/12)+7*k*sin(61*k*Pi/12)),k=0..infinity),18);

$0.0000412183133260668984 - 4.79642242946408952\cdot 10^{-24}$ I

The imaginary part of the above answer obtained by Maple is very small, so can be ignored.

4. Conclusion

From the above discussion, we know the geometric series and the integration term by term play significant roles in the theoretical inferences of this study. In fact, the applications of these two theorems are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

References

[1] M. L. Abell and J. P. Braselton, *Maple by Example*, 3rd ed., New York: Elsevier Academic Press, 2005.
[2] D. Richards, *Advanced Mathematical Methods with Maple*, New York: Cambridge University Press, 2002.
[3] F. Garvan, *The Maple Book*, London: Chapman & Hall/CRC, 2001.
[4] J. S. Robertson, *Engineering Mathematics with Maple*, New York: McGraw-Hill, 1996.
[5] C. T. J. Dodson and E. A. Gonzalez, *Experiments in Mathematics Using Maple*, New York: Springer-Verlag, 1995.
[6] R. J. Stroeker and J. F. Kaashoek, *Discovering Mathematics with Maple : An Interactive Exploration for Mathematicians, Engineers and Econometricians*, Basel: Birkhauser Verlag, 1999.
[7] C. Tocci and S. G. Adams, *Applied Maple for Engineers and Scientists*, Boston: Artech House, 1996.
[8] C. -H. Yu, " A study of the integrals of trigonometric functions with Maple," *Proceedings of IIE Asian Conference 2013*, National Taiwan University of Science and Technology, Taiwan, vol. 1, pp. 603-610, 2013.
[9] C. -H. Yu, " Using Maple to study the integrals of trigonometric functions," *Proceedings of 6th IEEE/International Conference on Advanced Infocomm Technology*, National United University, Taiwan, no. 00294, 2013.
[10] C.-H. Yu, " Application of Maple on evaluation of definite integrals," *Applied Mechanics and Materials*, 2013, in press.
[11] C.-H. Yu, " Application of Maple on the integral problems," *Applied Mechanics and Materials*, 2013, in press.
[12] C.-H. Yu, " Application of Maple on evaluating the closed forms of two types of integrals," *Proceedings of 17th Mobile Computing Workshop*, Chang Gung University, Taiwan, ID16, 2012.
[13] C.-H. Yu, " Application of Maple on some type of integral problem, " *Proceedings of Ubiquitous-Home Conference 2012*, Kun Shan University, Taiwan, pp.206-210, 2012.
[14] C.-H. Yu, " Application of Maple on some integral problems," *Proceedings of International Conference on Safety & Security Management and Engineering Technology 2012*, WuFeng University, Taiwan, pp. 290-294, 2012.
[15] C.-H. Yu, " Application of Maple: taking two special integral problems as examples," *Proceedings of 8th International Conference on Knowledge Community*, Chinese Culture University, Taiwan, pp.803-811, 2012.
[16] C.-H. Yu, " Application of Maple on the integral problem of some type of rational functions, " *Proceedings of Annual Meeting and Academic Conference for Association of IE 2012*, Da-Yeh University, Taiwan, D357-D362, 2012.
[17] T. M. Apostol, *Mathematical Analysis*, 2nd ed., Boston: Addison-Wesley, p269, 1975.