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A NOVEL PLRC-TLM FORMULATION FOR MODELLING DISPERSIVE MEDIA

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Abstract

The piecewise linear Recursive convolution (PLRC) technique has been developed and successfully integrated into the transmission line matrix (TLM) algorithm to model the electromagnetic wave propagation in dispersive media. The obtained results are in good agreement with their analytical counterparts.

Keywords: Transmission line matrix algorithm; piecewise linear Recursive convolution; electromagnetic wave; dispersive media.

1. Introduction

In recent years, there has been growing interest in computing propagation of waves through dispersive media using numerical methods [1-2]. Among these, the transmission line matrix (TLM) method is a powerful tool to deal with electromagnetic problems [3-5]. Based on the discrete model of Huygens's principle, TLM has been successfully introduced to handle dispersive media. Many techniques are integrated into the TLM algorithm. J. Paul et al [6, 7] integrate the Z-transform technique to formulate electric properties of dispersive media. Another approach using the constant recursive convolution (CRC) combined to voltage and current sources have been reviewed in [8-10]. In [11], the authors exploit the dependence between current density J and the electric field E to model a cold plasma slab. Recently a novel approach named Runge-Kutta Exponential Time Differencing technique (RKETD) has been developed and implemented for modeling anisotropic magnetized plasma media [12].

In this paper, a novel TLM algorithm based on the piecewise linear recursive convolution is developed and implemented to simulate electromagnetic wave interaction with cold plasma medium. The proposed model is validated by calculating the transmission and the reflection coefficient magnitudes of an electromagnetic plane wave through an air-plasma slab and compared to those obtained by the exact solution.

2. Formulation

For a cold plasma medium the following Maxwell's curl equation and constitutive relation are given by:

$$\nabla \times H = \frac{\partial D}{\partial t} \qquad (1)$$

$$D(t) = \varepsilon_0 \varepsilon_\infty E(t) + \varepsilon_0 \int_0^t \chi(\tau) E(t-\tau) d\tau \qquad (2)$$

$$\chi(t) = \frac{\omega_p^2}{\nu}[1 - \exp(-\nu t)]U(t) \qquad (3)$$

Where D is displacement field, χ is the susceptibly function of the medium.

In PLRC technique, we assume that the electric field E varies linearly over each time interval $[m\,\Delta t, (m+1)\Delta t]$. According to [13], E and D are expressed as follows:

$$E(n\Delta t - \tau) = E^{n-m} + \frac{E^{n-m-1} - E^{n-m}}{\Delta t}(\tau - m\Delta t) \qquad (4)$$

$$D^n = \varepsilon_0 \varepsilon_\infty E^n + \varepsilon_0 \sum_{m=0}^{n-1} (E^{n-m} \chi^m + \varepsilon_0 (E^{n-m-1} - E^{n-m}) \xi^m) \qquad (5)$$

Where

$$\chi^m = \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau)\, d\tau \qquad (6)$$

$$\xi^m = \frac{1}{\Delta t} \int_{m\Delta t}^{(m+1)\Delta t} (\tau - m\Delta t) \chi(\tau)\, d\tau \qquad (7)$$

By implementing (5) together with a similar expression for D^{n+1} into the discretization form of equation (1):

$$D^{n+1} - D^n = \Delta t\, (\nabla \times H)^{n+1/2} \qquad (8)$$

The electric field update equation is given as:

$$E^{n+1} = \frac{1}{(\varepsilon_\infty + \chi_0 + \xi_0)} \left[(\varepsilon_\infty - \xi_0) E^n + \psi^n + \frac{\Delta t}{\varepsilon_0} (\nabla \times H)^{n+1/2} \right] \qquad (9)$$

Where

$$\psi^n = \sum_{m=0}^{n-1}(E^{n-m}\,\Delta\chi^m + (E^{n-m-1} - E^{n-m})\Delta\xi^m) \tag{10}$$

$$\chi_0 = \frac{\omega_p^2}{\nu}\left[\Delta t - \frac{1}{\nu}(1 - \exp(-\nu\tau)\right] \tag{11}$$

And $$\xi_0 = \frac{\omega_p^2 \Delta t}{2\nu} + \frac{\omega_p^2}{\Delta t \nu^3}[(1 + \nu\Delta t)\exp(-\nu\tau) - 1] \tag{12}$$

Based on the equivalence between electromagnetic field (E, H) and the electric quantities (V, I) which governed by the following relations:

$$E = \frac{V}{\Delta l} \tag{13}$$

Where Δl is the TLM mesh width.

Equation (8) can be rewritten as:

$$V_u^{n+1} = \frac{1}{(\varepsilon_\infty + \chi_0 + \xi_0)}\left[(\varepsilon_\infty - \xi_0)V_u^n + \psi^n \Delta l + \frac{\Delta t \Delta l}{\varepsilon_0}\,(\nabla \times H)_u^{n+1/2}\right] \tag{14}$$

The application of charge conservation's laws to the symmetrical condensed node (SCN) which is characterized by (12 x 12) scattering matrix, three new stubs with admittances Y_{sx}, Y_{sy} and Y_{sz} are injected respectively into ports 13, 14 and 15 fed by voltage sources V_{sx}, V_{sy} and V_{sz}. These stubs model the electrical dispersive properties of the cold plasma medium. The total voltage in a TLM node is expressed as:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} = \begin{pmatrix} \frac{2}{4+Y_x}\left(V_1^i + V_2^i + V_{12}^i + V_9^i + Y_x V_{13}^i + 0.5\, V_{sx}\right) \\ \frac{2}{4+Y_y}\left(V_3^i + V_4^i + V_{11}^i + V_8^i + Y_y V_{14}^i + 0.5\, V_{sy}\right) \\ \frac{2}{4+Y_z}\left(V_5^i + V_6^i + V_{10}^i + V_7^i + Y_z V_{15}^i + 0.5\, V_{sz}\right) \end{pmatrix}^{n+1} \tag{15}$$

Where $$\begin{pmatrix} Y_x \\ Y_y \\ Y_z \end{pmatrix} = \begin{pmatrix} 4(\varepsilon_\infty + \chi_0 + \xi_0 - 1) \\ 4(\varepsilon_\infty + \chi_0 + \xi_0 - 1) \\ 4(\varepsilon_\infty + \chi_0 + \xi_0 - 1) \end{pmatrix} \tag{16}$$

The voltage sources (V_{sx}, V_{sy}, V_{sz}) are calculated recursively from the following expression:

$$V_{su}^{n+1} = -V_{su}^n - 4\chi_0 V_u^n - 4\psi^n \tag{17}$$

The implementation process of the PLRC-TLM algorithm is given as:

a- The recursive accumulator ψ^n is given by equation (10),

b- The update of the voltage sources is calculated from equation (17),

c- The total electric field on each node is deduced from equation (15) taking into account the equation (13).

3. Numerical results

In order to validate the PLRC-TLM schemes, we study the interaction of an electromagnetic Gaussian wave illuminating an air-cold plasma interface. The computational domain is subdivided into (1x1x1000) cells; each cell is 75µm, the plasma slab occupies the 200 cells in the middle with the physical parameters:

The electron collision frequency: ν=2π x50x3.18x109 rad/s.

The plasma frequency: ω_p=2πx47.7518x109 rad/s

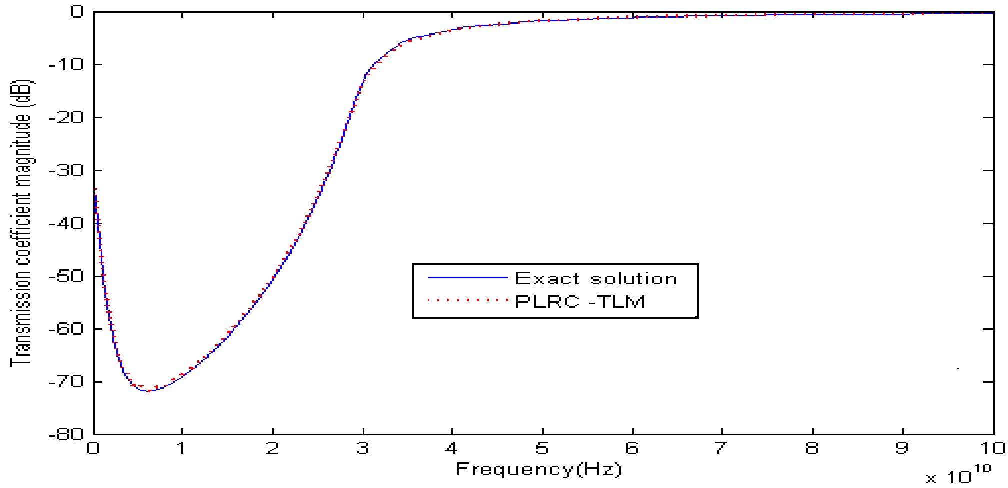

Figure 1: Transmission coefficient magnitude versus frequency.

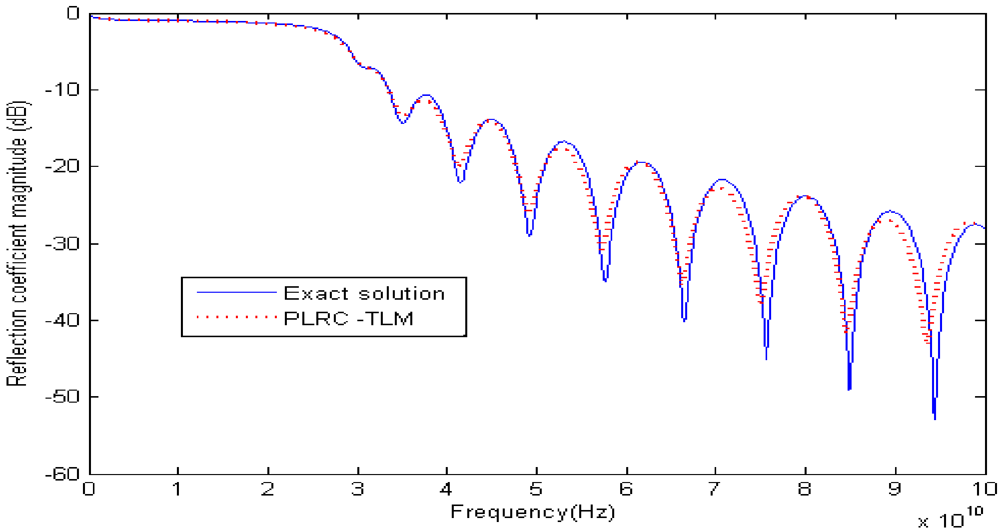

Figure 2: Reflection coefficient magnitude versus frequency.

Transmitted and reflected fields are stored during 20000 iterations, and converted to frequency domain through Inverse Fourier Transform. The magnitudes of transmission and reflection coefficients are computed and plotted respectively in figures 1 and 2. It can be observed that the PLRC-TLM scheme results are in good agreement with analytical solution.

4. Conclusion

In this paper, a novel piecewise linear recursive convolution-TLM technique is successfully developed and implemented for modelling dispersive cold plasma media. Results of the proposed model are in a good agreement with analytical solution.

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