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ADE-TLM Scattered-Field Formulation for Kerr and Raman Nonlinear Dispersive Media

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Abstract

In this paper, we propose a scattered-field formulation for modelling Kerr and Raman nonlinear dispersive media effects using the Transmission Line Matrix method with the symmetrical condensed node (SCN -TLM). This model is based on novel voltage sources and the Auxiliary Differential Equation (ADE) technique.

Keywords: Auxiliary Differential Equation (ADE) technique, Transmission Line Matrix (TLM), Lorentz, Kerr, Raman.

1. Introduction

Several differential numerical methods are implemented, in temporal domain, to study dispersive nonlinear media. Based on different techniques of discretization of own equations, the developed approaches have efficiently simulated nonlinear dispersive media. These approaches include the linear and nonlinear properties and take into account physical complexity of media such as Kerr effects, Raman diffusion and/or photonic absorption. The Finite-Difference Time-Domain (FDTD) method and the numerical resolution of polarization and Maxwell's equations are often used to characterize intrinsically nonlinear media, and to predict the behavior of propagated electromagnetic (EM) waves [1]-[3]. The recent developments of the transmission line matrix (TLM) method have shown its flexibility and efficiency for the simulation of EM waves propagation in complex media. This technique has been able to simulate EM wave propagation in dispersive anisotropic nonlinear media [4]-[6].

In this paper, we use the TLM numerical method to model the linear and nonlinear dispersion properties. A novel algorithm based on the TLM method with condensed symmetrical node (SCN) and new voltage sources combined with the auxiliary differential equation ADE, is used to model the Lorentz dispersion, the nonlinear instantaneous Kerr and the retarded Raman effects in nanostructured nonlinear optical media.

Numerical results that demonstrate the validity and efficiency of the proposed algorithm TLM-ADE as well as the nonlinear formulation of the electric field are presented.

Excellent accordance is achieved between the ADE-TLM calculated and exact theoretical results for the reflection coefficient in Lorentz, Kerr and Raman media.

2. Formulation

The Maxwell-Ampere equation for an EM wave propagating along the z-direction (i.e., E_x component) in a dispersive nonlinear medium can be written as follows:

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$$\frac{\partial \bar{E}_x(t)}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_\infty}\left[-\frac{\partial \bar{P}_x(t)}{\partial t} + \left(\nabla \Lambda \bar{H}\right)_x\right] \quad (1)$$

Where ε_0 is the dielectric constant of free space, ε_∞ is the relative dielectric constant in the limit of infinite frequency and $P_x(t) = P_L(t) + P_{NL}(t)$ is the polarization term. The linear part $P_L(t) = P_{Lorentz}(t)$ describes the linear Lorentz dispersive properties of the medium and the nonlinear one $P_{NL}(t) = P_{Kerr}(t) + P_{Raman}(t)$ represents the instantaneous Kerr and retarded Raman effects. E_x and P_x are respectively the components, along the x-axis, of the electric field and polar ization vectors.

Using centered differencing scheme between time steps $t^{n+1} = (n+1)\Delta l$ and $t^n = n\Delta l$, we can write the discretized equation for the electric and polarization fields in regular space $(\Delta x = \Delta y = \Delta z = \Delta l)$ as follows:

$$E_x^{n+1} = E_x^n - \frac{1}{\varepsilon_0 \varepsilon_\infty}(P_x^{n+1} - P_x^n) + \frac{\Delta t}{\varepsilon_0 \varepsilon_\infty}\left(\nabla \Lambda H^{n+\frac{1}{2}}\right)_x \quad (2)$$

To develop a TLM model for the numerical treatment of these problems we need to transform the EM field parameters into circuit parameters that are related to transmission lines. Substituting in (2), the component E_x by its electric equivalent $\frac{V_x}{\Delta l}$, we obtain the expression of the discretized electric voltage component V_x^{n+1} along the x-axis at time step t^{n+1} :

$$V_x^{n+1} = V_x^n - \frac{\Delta l}{\varepsilon_0 \varepsilon_\infty}(P_x^{n+1} - P_x^n) + \frac{\Delta t \Delta l}{\varepsilon_0 \varepsilon_\infty}\left(\nabla \Lambda H^{n+\frac{1}{2}}\right)_x \quad (3)$$

The proposed ADE-TLM algorithm, unifies all types of dispersion (of the second and the third order) of medias expressed by the polarization terms appearing in (3), in a single formula expressing the component of the electric voltage along the z-direction.

In the sequel, we give the numerical expressions of the polarization of the second and the third order ($P_L(t)$ and $P_{NL}(t)$) in terms of electric voltage.

A. *Linear Lorentz polarization*

The Lorentz polarization as a function of electric voltage is governed by the following auxiliary differential equation:

$$\frac{\partial^2 P_{Lorentz}(t)}{\partial t^2} + 2\delta_{Lorentz}\frac{\partial P_{Lorentz}(t)}{\partial t} + \omega_{Lorentz}^2 P_{Lorentz}(t) = \frac{\varepsilon_0(\varepsilon_{Lorentz} - \varepsilon_\infty)}{\Delta l}\omega_{Lorentz}^2 V(t), \quad (4)$$

Where $\omega_{Lorentz}$ is the Lorentz characteristic resonant frequency, $\delta_{Lorentz}$ the damping factor and $\varepsilon_{Lorentz}$ the static permittivity caused by the Lorentz dispersion.

By using centered differencing, the polarization of the Lorentz dispersion $P_{Lorentz}^{n+1}$ at time step t^{n+1} obeys to the following difference equation:

$$P_{Lorentz}^{n+1} = a_{Lorentz}P_{Lorentz}^n + b_{Lorentz}P_{Lorentz}^{n-1} + c_{Lorentz}\frac{V_x^n}{\Delta l} \quad (5)$$

Where

$$a_{Lorentz} = \frac{2 - \omega_{Lorentz}^2 \Delta t^2}{1 + \delta_{Lorentz}\Delta t}$$

$$b_{Lorentz} = -\frac{1 - \delta_{Lorentz}\Delta t}{1 + \delta_{Lorentz}\Delta t}$$

$$c_{Lorentz} = \frac{\varepsilon_0(\varepsilon_{Lorentz} - \varepsilon_\infty)\omega_{Lorentz}^2 \Delta t^2}{1 + \delta_{Lorentz}\Delta t}$$

B. *Nonlinear Kerr and Raman polarization*

The polarization caused by the instantaneous Kerr nonlinearity is written, as function of the electric voltage by:

$$P_{Kerr}(t) = \frac{\varepsilon_0 \chi_0^{(3)}}{\Delta l} V(t) \int_{-\infty}^{t} \frac{\alpha \delta(t - t')}{\Delta l^2} V^2(t')dt' = \frac{\alpha \varepsilon_0 \chi_0^{(3)}}{\Delta l^3} |V^2(t)| V(t) \tag{6}$$

$\chi_0^{(3)}$ being the third order nonlinear susceptibility and α, $0 \le \alpha \le 1$, represents the relative strengths of the Kerr and Raman polarizations.

Analogous to the Lorentz case, the polarization of the Kerr dispersion P_{Kerr}^{n+1} at time step t^{n+1} can be written as follows:

$$P_{Kerr}^{n+1} = \frac{\alpha \varepsilon_0 \chi_0^{(3)}}{\Delta l^3} |V_x^{n+1}|^2 V_x^{n+1} \tag{7}$$

As a convolution, the polarization caused by the Raman effect may be writing as:

$$P_{Raman}(t) = \frac{\varepsilon_0}{\Delta l} V(t) \chi_{Raman}^{(3)}(t) * \frac{V^2(t)}{\Delta l^2} \tag{8}$$

To solve (8), we introduce an auxiliary variable $S(t)$ for the convolution:

$$S(t) = \chi_{Raman}^{(3)}(t) * \frac{V^2(t)}{\Delta l^2} \tag{9}$$

Then we apply the Fourier transform to (9):

$$\tilde{S}(\omega) = \tilde{\chi}_{Raman}^{(3)}(\omega) * \Im\left[\frac{V^2(t)}{\Delta l^2}\right] \tag{10}$$

Where $\tilde{S}$ and $\tilde{\chi}$ denote the Fourier transform s of the corresponding time functions.
The retarded response function is:

$$\tilde{\chi}_{Raman}^{(3)}(\omega) = \frac{(1-\alpha)\chi_0^{(3)} \omega_{Raman}^2}{\omega_{Raman}^2 + 2j\omega\delta_{Raman} - \omega^2} \tag{11}$$

With

$$\omega_{Raman} = \sqrt{\frac{\tau_1^2 + \tau_2^2}{\tau_1^2 \tau_2^2}}$$

$$\delta_{Raman} = \frac{1}{\tau_2}$$

Next insert (11) into (10), multiply by ($\omega_{Raman}^2 + 2j\omega\delta_{Raman} - \omega^2$) and transform to the time-domain functions by $j\omega \to \frac{\partial}{\partial t}$ and $\omega^2 \to -\frac{\partial^2}{\partial t^2}$ to get the following auxiliary differential equation:

$$\frac{\partial^2 S(t)}{\partial t^2} + 2\delta_{Raman} \frac{\partial S(t)}{\partial t} + \omega_{Raman}^2 S(t) = \frac{(1-\alpha)\chi_0^{(3)} \omega_{Raman}^2}{\Delta l^2} V_x^2(t) \tag{12}$$

The finite difference expression for the time derivative of (12) centered at time step t^{n+1} is then given by:

$$S^{n+1} = a_{Raman} S^n + b_{Raman} S^{n-1} + \frac{c_{Raman}}{\Delta l^2} V_x^{n2} \tag{13}$$

With

$$a_{Raman} = \frac{2-\omega_{Raman}^2 \Delta t^2}{1+\delta_{Raman}\Delta t}$$

$$b_{Raman} = -\frac{1-\delta_{Raman}\Delta t}{1+\delta_{Raman}\Delta t}$$

$$c_{Raman} = \frac{(1-\alpha)\chi_0^{(3)}\omega_{Raman}^2 \Delta t^2}{1+\delta_{Raman}\Delta t}$$

Finally, the discretized Raman polarization P_{Raman}^{n+1} at time step t^{n+1} can be written as follows:

$$P_{Raman}^{n+1} = \frac{\varepsilon_0}{\Delta l} V_x^{n+1} S^{n+1} \tag{14}$$

3. Update of the Constitutive Equation

By substituting the terms of the polarization given by (5), (7) and (14) in (3), we can write the discretized nonlinear relation describing electrical voltage at time step t^{n+1} as:

$$V_x^{n+1} = V_x^n - \frac{\Delta l}{\varepsilon_0 \varepsilon_\infty}(P_{Lorentz}^{n+1} - P_{Lorentz}^n) - \frac{\alpha\chi_0^{(3)}}{\Delta l^2 \varepsilon_\infty}\left(|V_x^{n+1}|^2 V_x^{n+1} - |V_x^n|^2 V_x^n\right) - \frac{1}{\varepsilon_\infty}(V_x^n S^{n+1} + V_x^n S^n) + \frac{\Delta t \Delta l}{\varepsilon_0 \varepsilon_\infty}\left(\nabla \Lambda H^{n+\frac{1}{2}}\right)_x \tag{15}$$

Writing the magnetic field components of (15) in terms of incident and reflected pulses [7]-[8], then applying the charge conservation principle for the symmetrical condensed node SCN with a capacitive stub of normalized admittance $Y_{ox}(t)$ we obtain the following expression for the discretized electric voltage V_x^{n+1} at time step t^{n+1}:

$$V_x^{n+1} + \frac{A_2}{A_1}|V_x^{n+1}|^2 V_x^{n+1} = \frac{2}{4+Y_{ox}}_{n+1}\left(V_1^i + V_2^i + V_9^i + V_{12}^i + Y_{ox} V_{13}^i + 0.5 V_{sx}\right) \tag{16}$$

Where

$$A_1 = \varepsilon_\infty + S^{n+1}$$

$$A_2 = \frac{\alpha\chi_0^{(3)}}{\Delta l^2}$$

V_{sx} and Y_{ox} are respectively the voltage source, and the normalized admittance terms in order to take into account, in the TLM mesh, linear and nonlinear dispersive properties and also Kerr effects and Raman interactions in the nonlinear medium. They are given by:

$$V_{sx} = -{}_n V_{sx} + 4\left[\left(\varepsilon_\infty + S^{n+1} - A_1\right) V_x^n + 2A_2 |V_x^n|^2 V_x^n - \frac{\Delta l}{\varepsilon_0}\left(P_{Lorentz}^{n+1} - P_{Lorentz}^n\right)\right] \tag{17}$$

$${}_{n+1}Y_{ox} = 4\left(\varepsilon_\infty + S^{n+1} - 1\right) \tag{18}$$

The ADE-TLM model with voltage sources is based on recursive calculation of normalized admittance made in (18) and the voltage sources expressed in (17). The obtained values are then inserted in (16). The solution of the last equation is then used in the calculation of reflected pulses and in the connection process along the TLM mesh nodes.

4. Numerical results

In order to show the efficiency of the new model ADE-TLM with voltage sources, we present spatial variations of electrical field propagation in a medium having a Lorentz linear dispersion characterized by the following [9]:

$$\varepsilon_s = 5.25, \varepsilon_\infty = 2.25, \ \omega_{Lorentz} = 0.4\times10^{15} \text{rad/s} \ \text{and} \ \delta_{Lorentz} = 0.1\omega_{Lorentz}.$$

Further, the medium nonlinearity is assumed to be characterized by [9]:

$$\chi^{(3)} = 7\times10^{-2}(\text{V/m})^{-2}, \tau_1 = 12.2\,\text{fs}, \ \tau_2 = 32\,\text{fs} \ \text{and} \ \alpha = 0.7.$$

The considered TLM mesh dimensions are (1, 1, 5000) Δl with space step $\Delta l = 8$ nm. The air-nonlinear dispersive medium interface is located at $z = 8\Delta l$ and excited by Gaussian unit amplitude pulse. Recall that when an EM wave propagates in a medium, the field induces a time varying dipole moment in the individual atoms that comprise the medium. The oscillating atoms lose energy through radiative and nonradiative mechanisms. The implied polarization is expressed through changes in the reflected field.

Fig. 1 illustrates the time evolution of the reflected electric field in Lorentz, Kerr and Raman medium. Also the dominant aspect of the nonlinear Kerr and Raman effects relative to those of Lorentz in the reflected field in the nonlinear medium is shown in Fig. 1.

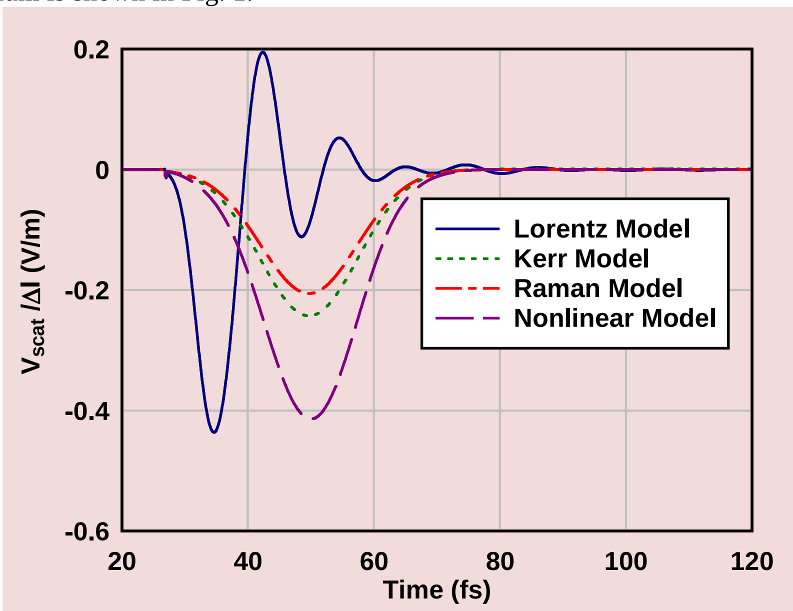

Fig. 1. Time evolution of the reflected electric field in Lorentz, Kerr, Raman and nonlinear media located at $z = 18\Delta l$.

Fig. 2 compares the ADE-TLM calculated and exact theoretical results [10] for the magnitude of the reflection coefficient for Lorentz, Kerr and Raman media.

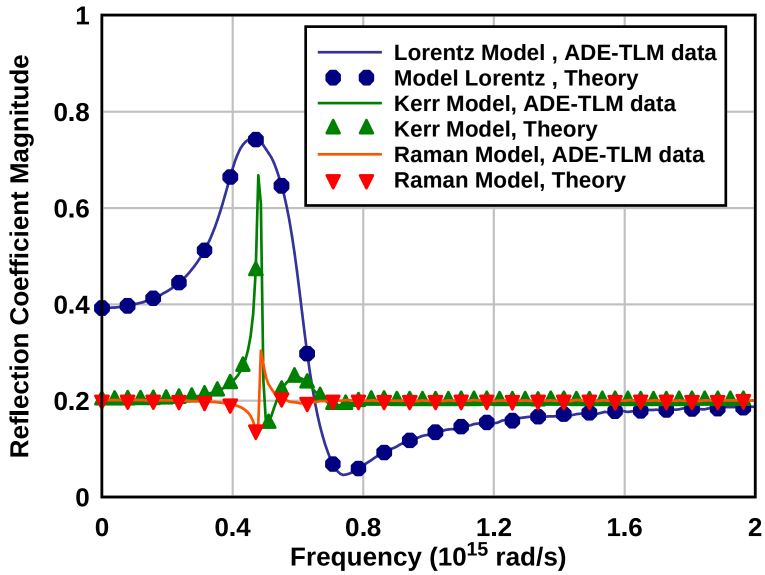

Fig. 2. compares the ADE-TLM calculated and exact theoretical results for the magnitude of the reflection coefficient for Lorentz, Kerr and Raman media..

5. CONCLUSION

We have efficiently analyzed Maxwell's equations and different polarization expressions for modeling propagation of reflected optical pulses in nonlinear dispersive media. The ADE-TLM model we introduced a voltage sources modeling linear and nonlinear properties and the variable admittance concept.

An excellent agreement was obtained between the model ADE-TLM and the theoretical results.

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